Probing RNN Encoder-Decoder Generalization of Subregular Functions Using Reduplication

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Talk in a Nutshell

- Formal Languages/Automata:
  - **Necessary and sufficient conditions** on computable functions
  - Provide target function classes for generalization/learning
  - transparent, analytical guarantees independent of the machine
- Recurrent Neural Network/ finite-state connections
- What is the generalization capacity of RNN Encoder-Decoders?

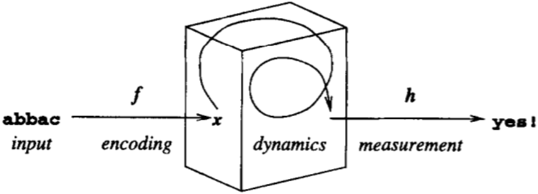
Encoder-decoders and Subregular Reduplication

- Reduplication: variable-length subregular copy functions
- Vanilla Encoder-Decoders struggle to capture generalizable reduplication, networks with attention reliably succeed
- Attention weights mirror subregular 2-way FST processing, suggests they are approximating them

RNN and regular languages

Language: *Does string w belong to stringset (language) L*

- Computed by different classes of grammars (**acceptors**)



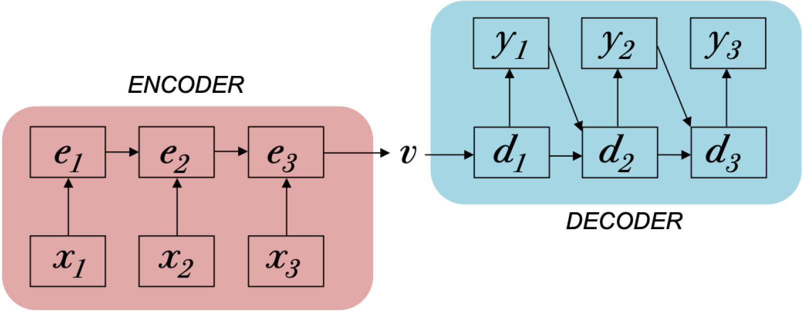
How expressive are RNNs?

Turing complete	infinite precision+time	(Siegelmann, 2012)
$\subseteq$ counter languages	LSTM/ReLU	(Weiss et al., 2018)
Regular	SRNN/GRU	(Weiss et al., 2018)
	asymptotic acceptance	(Merrill, 2019)
Weighted FSA	Linear 2nd Order RNN	(Rabusseau et al., 2019)
Subregular	LSTM problems	(Avcu et al., 2017)

pic credit: Casey 1996

RNN Encoder-Decoder and Transducers

- **Function**: *Given string w, generate* $f(w) = v$
  - = accepted pairs of input & output strings
  - Computed by different classes of grammars (**transducers**)
- Recurrent encoder maps a sequence to $v \in \mathbb{R}^n$, recurrent decoder language model conditioned on v (Sutskever et al., 2014)
- How expressive are they?



Brief typology of reduplication

- Reduplication is typologically common[1]
- Basic division: partial vs. total reduplication

(1) Partial reduplication = bounded copy

a.	CV:	guyon → gu~guyon 'to jest'→'to jest repeatedly'	(Sundanese)
b.	Foot:	(gindal)ba → gindal~gindalba 'lizard sp.' → 'lizards'	(Yidin)
c.	Syllable	vam.se → vam~vamse 'hurry' → 'hurry (habitual)'	(Yaqui)

(2) Total reduplication = unbounded copy

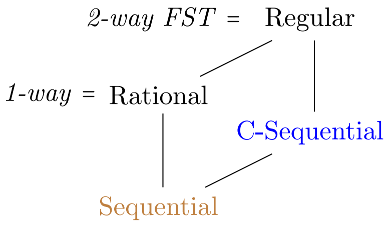
a.		wanita→wanita~wanita 'woman'→'women'	(Indonesian)

[1] (Moravcsik, 1978; Rubino, 2013)

Subregular computing of reduplication

- Why reduplication (Red)?
  - inhabits **sub**classes of **regular** string-to-string functions
  - computed by restricted types of **Finite-State Transducers**

1. **1-way FST**: reads input once in one direction
   ~ computes Rational functions
   e.g., Sequential functions like partial Red
2. **2-way FST**: reads multiple times, moves back and forth
   ~ computes Regular functions
   e.g., Concatenated-Sequential functions like partial & total Red



Partial reduplication with 1-way FSTs

- Working example: pat→[pa~pat]

Partial reduplication with 1-way FSTs

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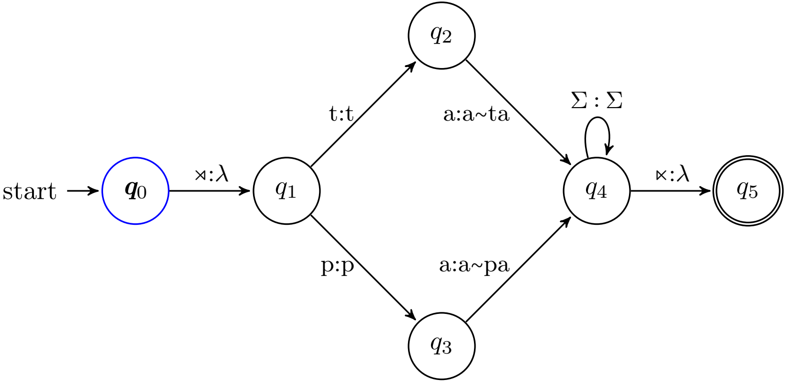
  Input: ⋊ p a t ⋉

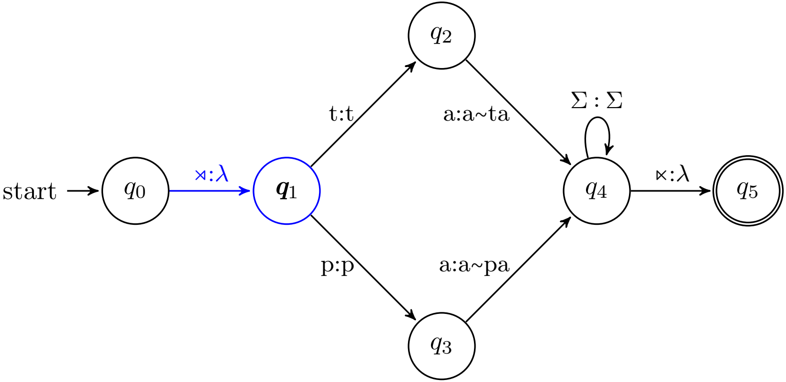
  Output:



Partial reduplication with 1-way FSTs

- Working example: pat→[pa~pat]

Input:	$\rtimes$	p	a	t	$\ltimes$
Output:					



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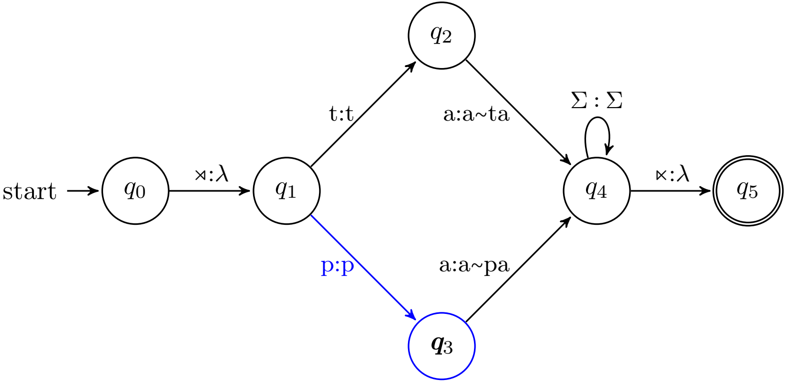

Partial reduplication with 1-way FSTs

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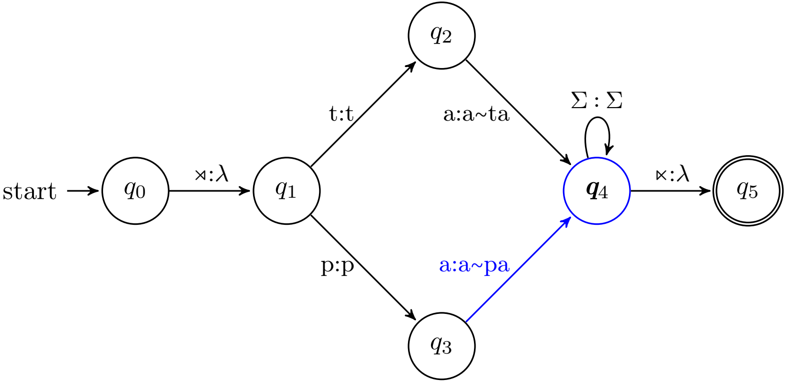
Input:	⋊	p	a	t	⋉
Output:		p	a~pa		



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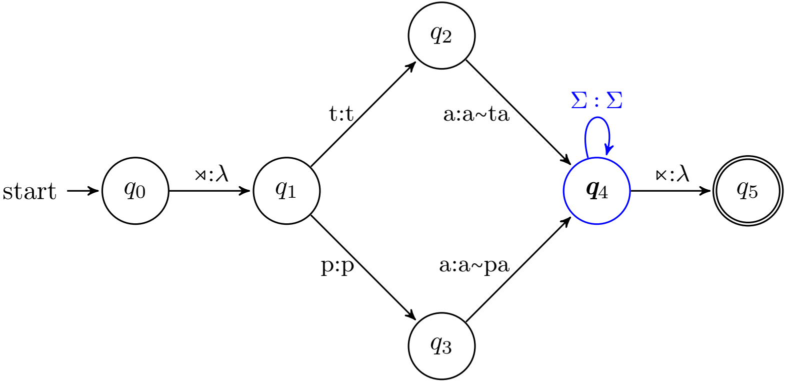

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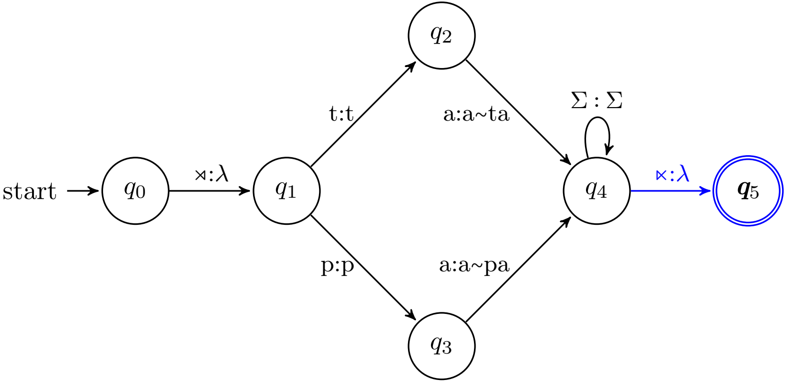

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Input:	⋊	p	a	t	⋉	☺
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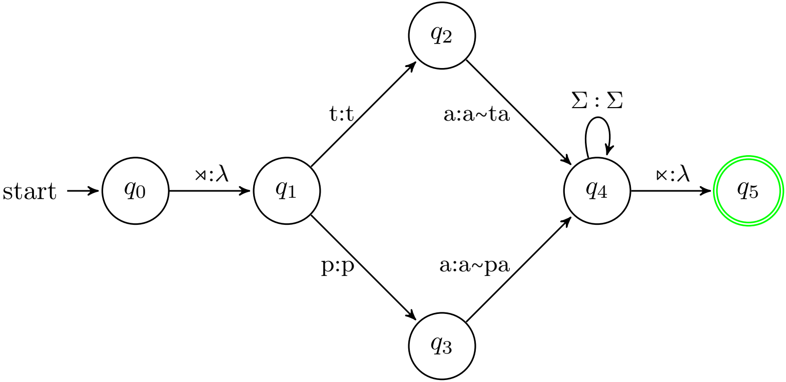

1-way FST Limitations

- How does a 1-way FST handle reduplication?
  - → memorizes all possible reduplicants
- Many limitations:

1. **State explosion**:
   - scaling problems as size of reduplicant and alphabet increases
   - unwieldy machines (Roark and Sproat, 2007:54)
2. **Limited expressivity**:
   - can do partial reduplication but not total reduplication
   - No bound on how big the copies are
3. **Segment alignment**:
   - Memorizes, doesn't 'copy'

Partial reduplication with 2-way FSTs

- Working example: pat→[pa~pat]

Partial reduplication with 2-way FSTs

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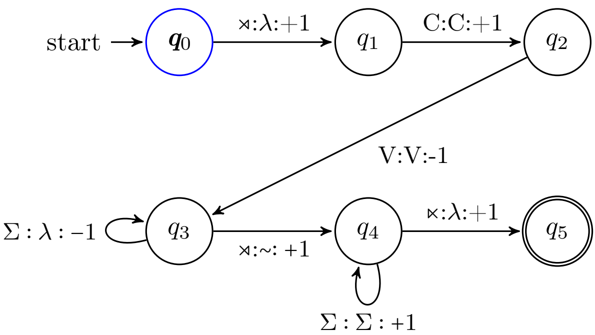

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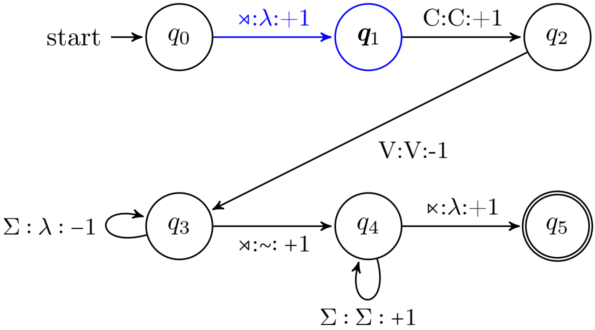

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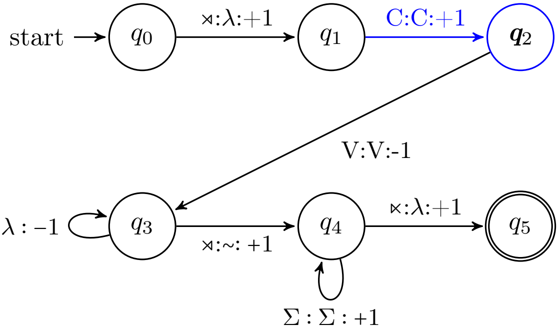

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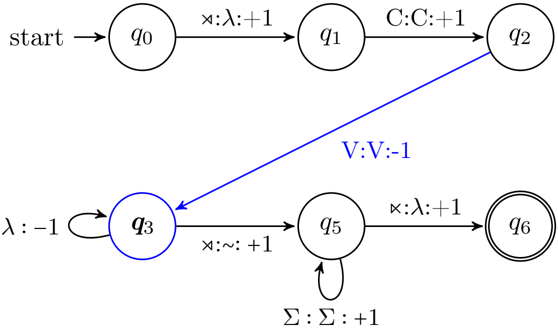

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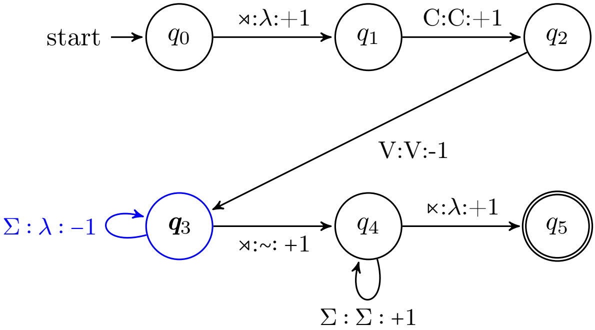

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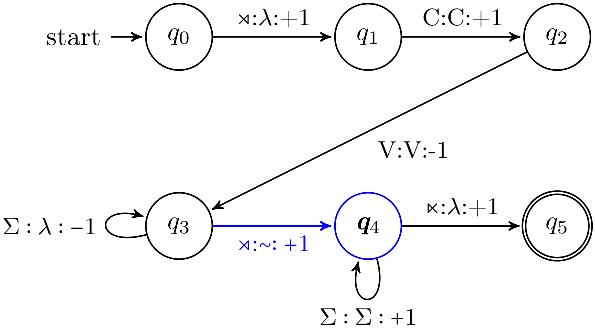

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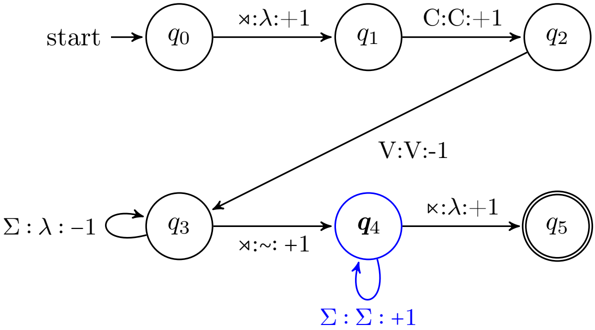

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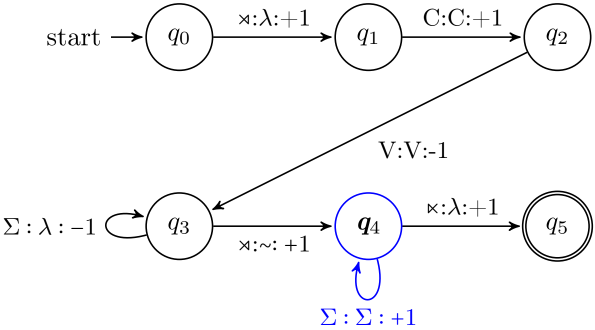

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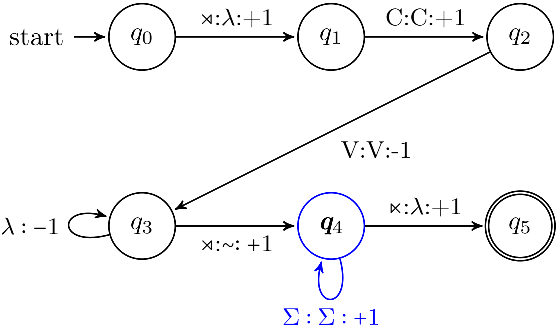

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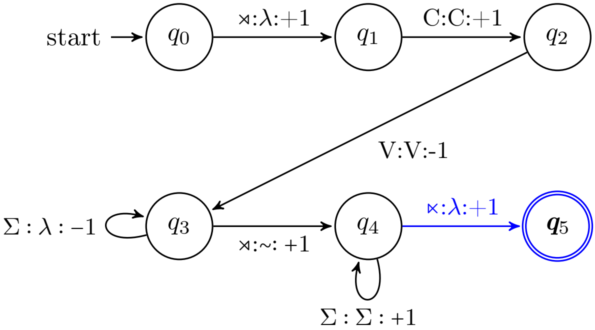

Partial reduplication with 2-way FSTs

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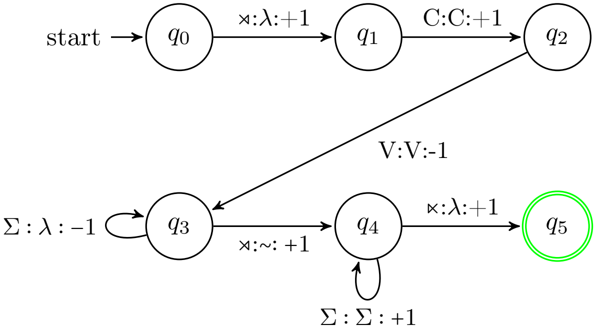


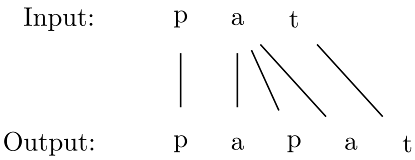
Reduplication with 2-way FSTs

- How does 2-way FST handle reduplication?
  → look *back* at the input to generate copies
- Increased expressivity, removes limitations...

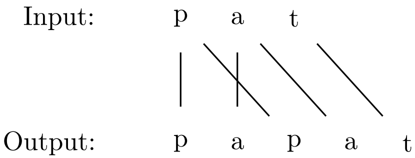
1. **Compact**:
   - no state explosion
2. **Expressive**:
   - can do partial and total reduplication
3. **Segment alignment**:
   - Output segments are aligned with the ‘right’ input segments
   - Formally, look at *origin semantics* of how input-output segments align (Bojańczyk, 2014)

Segment alignment with FSTs

- **Origin information**: origin of output symbols in the input
- 1-way FSTs remember what to repeat, they don't actively copy



- But linguistic theory says "copy" like a 2-way FST!



Learning Reduplication

Reduplication is *provably* learnable in polynomial time and data (Chandlee et al., 2015; Dolatian and Heinz, 2018)

RNNs with segmental inputs cannot be trained as reduplication acceptors (Gasser, 1993; Marcus et al., 1999)

- Recognizing reduplication requires the comparison of static subsequences - difficult for an RNN to store

Encoder-Decoders learn reduplication with a fixed-size reduplicant in a small toy language (Prickett et al., 2018)

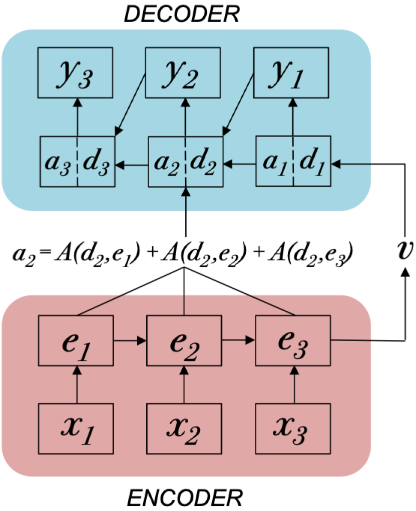
- Generalizable to novel segments and sequences
- Generalization to novel lengths not tested, computable by 1-way FST that uses featural representations

Recurrence

- **Recurrence relation:** The function relating hidden states in the encoder and decoder RNNs - affects practical expressivity of network
- Two types of recurrence tested:
  - **sRNN** - t^{th} state is a nonlinear function of the t^{th} input and state $t-1$ (Elman, 1990)
  - **GRU** - t^{th} state is a linear function of three functions (gates) of the t^{th} input and state $t-1$ (Cho et al., 2014)
- Saturating nonlinearities ($tanh$) - sRNNs and GRUs cannot count with finite precision (Weiss et al., 2018)
- LSTM is supra-regular, we are testing necessary properties of RNN and GRU, which are finite-state (Merrill, 2019)

Attention

- In standard ED, the encoded representation is the only link between the encoder and decoder
- **Global attention** allows the decoder to selectively pull information from hidden states of the encoder (Bahdanau et al., 2014)
- **FLT Analog**: 2-way FST has full access to the input by moving back and forth



Test data

- Input-output mappings generated with 2-way FSTs from RedTyp database[2]
  1. Initial-CV — tasgati→ta~tasgati
     Fixed-size reduplicant
  2. Initial two-syllable (C*VC*V) — tasgati→tasga~tasgati
     Onset maximizing, fixed over vowels
  3. Total — tasgati→tasgati~tasgati
     Variably sized reduplicant
- 10,000 generated for each language, 70/30 train/test split
- Minimum string length 3 - maximum string length varied
- Alphabet of 10, 16, or 26 characters
- Boundary symbols (~) are not present

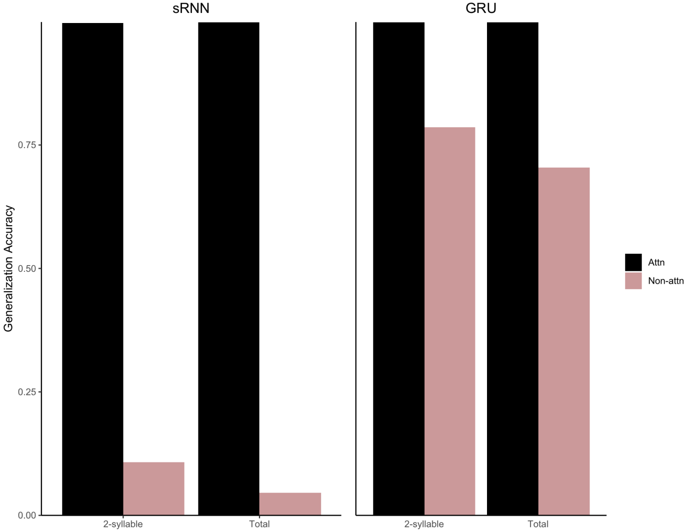
[2] Dolatian and Heinz (2019); also available on GitHub

Experiment 1

- Interaction between reduplication type, recurrence, and attention
  - Total and partial (two-syllable) reduplication
  - sRNN and GRU with and without attention
- Max string length: 9
- 10 symbols alphabet

Attention should improve function generalization across reduplication types and recurrence relations

Experiment 1

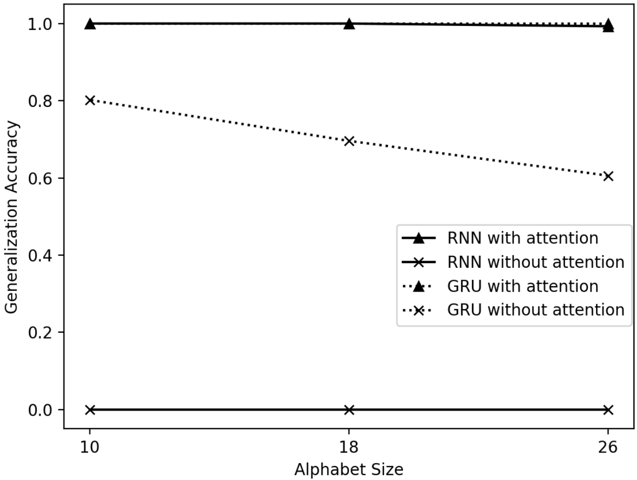


Experiment 2

- Effects of alphabet size and range of permitted string lengths
- CV reduplication only
- sRNN/GRU × attention/non-attention × 3 alphabet sizes × 7 length ranges

Network generalization while learning a general reduplication function should be invariant to language composition

Experiment 2



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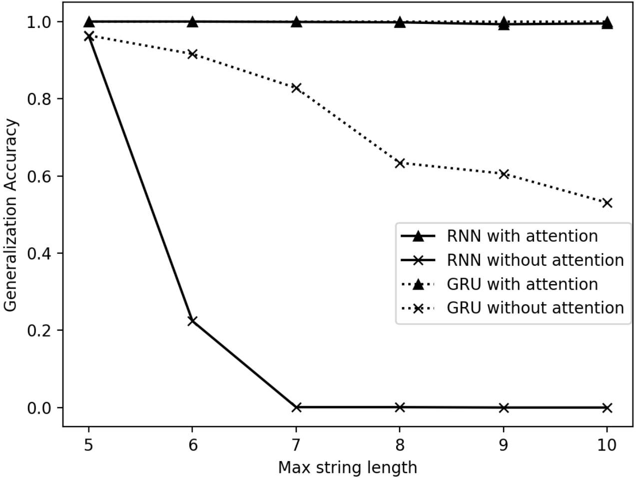
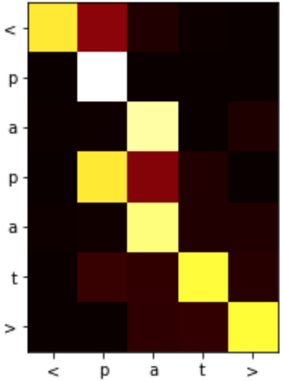


Discussion

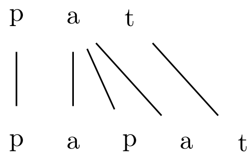
- Networks with global attention learn and generalize all types of reduplication and seem robust to string length and alphabet size
- sRNNs without attention show slightly better generalization of partial reduplication than total reduplication
  - Confound with less attested reduplicant lengths or a bias preferring the regular pattern?
- GRUs perform better than sRNNs across all conditions
  - Without attention not robust to length/alphabet - likely learning heuristics that capture most data rather than a general function

Networks that cannot see material in the input multiple times cannot learn generalizable reduplication

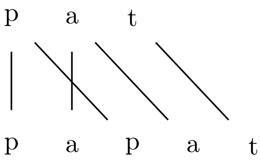
Attention and Origin Semantics

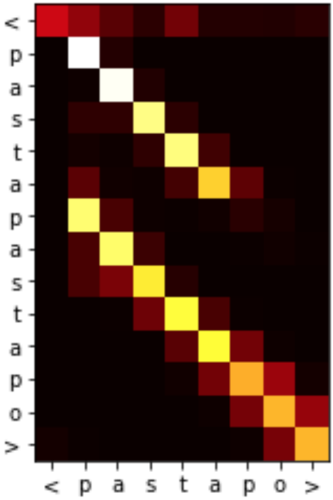


1-Way:



2-Way:

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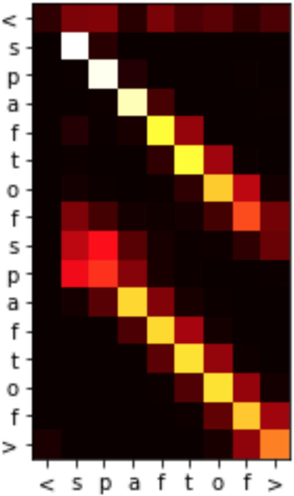
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Summary

1. **Why use reduplication functions?**
   - properties define fine-grained subregular function classes
   - Allows us to test the generalization capacity of neural nets
2. **Expressivity of attention**
   - Attention is necessary and sufficient for robustly learning and generalizing reduplication functions using Encoder-Decoders
3. **FST approximations**
   - Non-attention networks are limited to a single input pass, approximating 1-way FST
   - Attention networks can read the input again during decoding, approximating 2-way FST,
4. **Attention weights and origin information**
   - Evidence for approximation comes from attention weights
   - IO correspondence relations mirror origin semantics of 2-way FST
5. **Next step**: trying more copying and non-copying functions

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Guide to appendix

- Reduplication across FSTs and RNNs [25]
- Harmony Extensions [26]
- Finite-State Automata & Representation Learning [27]
- Learning Reduplication [28]
- Problems with 1-way FSTs for Total Reduplication [29]
- Total reduplication with 2-way FSTs [31]

Reduplication across FSTs and RNNs

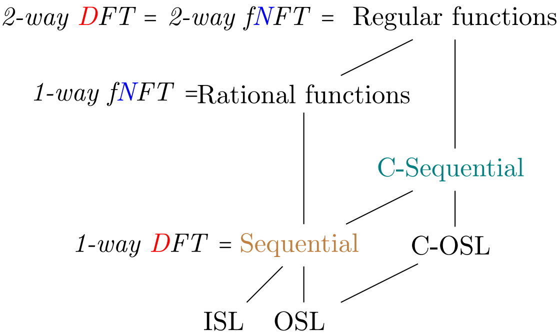
- 1-way and 2-way FSTs compute reduplicative functions differently

	1-way	2-way
Strategy?		
How does it reduplicate?	Memorize	Look back
Scaling?		
Is there state explosion	✓ ☹	✗ ☺
Expressive?		
Can it do total reduplication?	✗ ☹	✓ ☺
Alignment?		
Does origin information match theory?	✗ ☹	✓ ☺

- **Strategy** creates all additional properties
- **Link to RNNs** :
  - attention-less EDs compute like 1-way FSTs!
  - attention-based EDs compute like 2-way FSTs

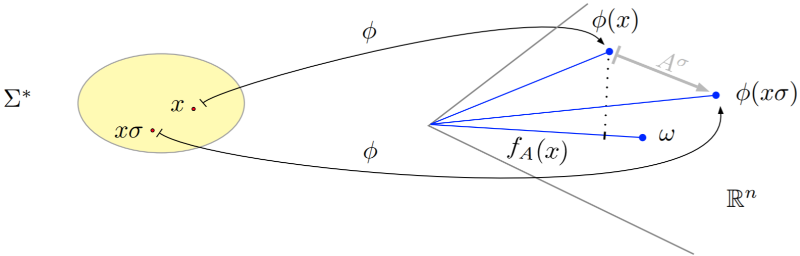
Next: Attention, 2-way, and Determinism

The subregular hierarchy is more subtle



- Does attention enable non-regularity? Non-determinism?
  - What about $w \to w^3$, $w \to ww^r$, $w \to w^w$, ...
- Idea: Use Harmony processes (Heinz and Lai, 2013)
  - harmony spans subregular hierarchy
  - unattested non-regular harmony (ex. Majority Rules)

Finite-State Automata & Representation Learning



- An FSA induces a mapping $\phi : \Sigma^* \to \mathbb{R}$
- The mapping ϕ is compositional
- The output $f_A(x) = \phi(x), \omega$ is linear in $\phi(x)$

Pic credit: Guillaume Rabusseau

Learning Reduplication

- Reduplication is *provably* learnable in polynomial time and data (Chandlee et al., 2015; Dolatian and Heinz, 2018)
- RNNs with segmental inputs cannot be trained as reduplication acceptors (Gasser, 1993; Marcus et al., 1999)
  - Recognizing reduplication requires the comparison of static subsequences - difficult for an RNN to store
- Encoder-Decoders learn reduplication with a fixed-size reduplicant in a small toy language (Prickett et al., 2018)
  - Generalizable to novel segments and sequences
  - Generalization to novel lengths not tested, computable by 1-way FST that uses featural representations

Problems with 1-way FSTs for Total

- 1-way FSTs can do Partial Red **inelegantly**
- Total reduplication **cannot** be modeled at all.
- **Why?**
  - copied portion has unbounded size
  - 1-way FST can't do that!
  - needs an infinite # of states

Problems with 1-way FSTs for Total

- Total reduplication **cannot** be modeled at all.
- **Can you approximate?**
  - some finite-state approximations exist...[3]
  - **But**: they impose un-linguistic restrictions (e.g. a finite bound on word size,...) so don't directly capture reduplication
- **Give up on finite-state?**
  - MCFGs, HPSG, pushdown accepters with queues[4]
  - But... those are recognizers not transducers

[3] Hulden (2009); Beesley and Karttunen (2003); Walther (2000)
[4] Albro (2005); Crysmann (2017); Savitch (1989)

Total reduplication with 2-way FSTs

- Total reduplication copies an unbounded size

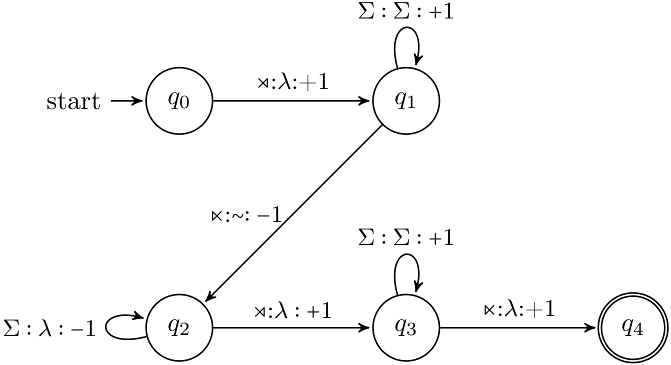
(3) wanita→wanita~wanita 'woman'→'women' (Indo.)

Total reduplication with 2-way FSTs

- Total reduplication copies an unbounded size

(4) wanita→wanita~wanita 'woman'→'women' (Indo.)

- This 2-way FST reads the input left to right (+1), goes back (-1), and reads the input again (+1)



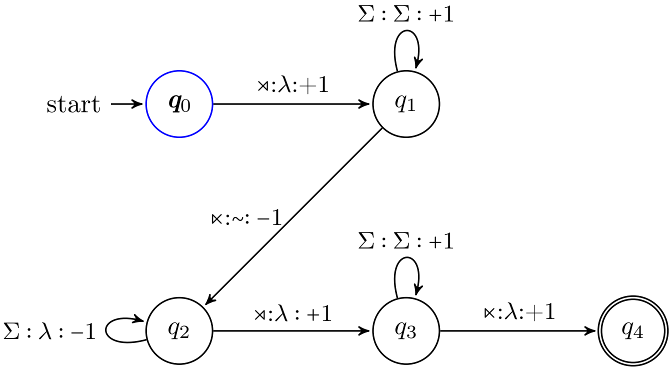
Total reduplication with 2-way FSTs

- Indonesian example: wanita→wanita~wanita
- Working example: bye→?

Total reduplication with 2-way FSTs

- Indonesian example: wanita→wanita~wanita
- Working example: bye→bye~bye

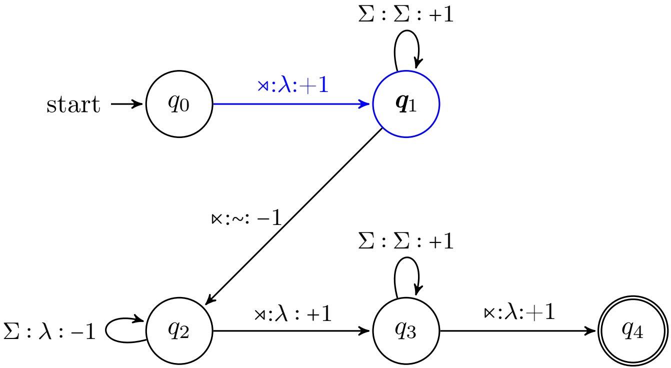
Input: ⋊ b y e ⋉
Output:



Total reduplication with 2-way FSTs

- Indonesian example: wanita→wanita~wanita
- Working example: bye→bye~bye

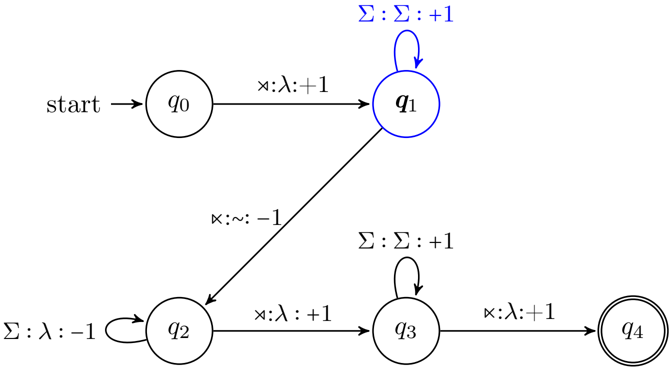
Input:	⋊	b	y	e	⋉
Output:					



Total reduplication with 2-way FSTs

- Indonesian example: wanita→wanita~wanita
- Working example: bye→bye~bye

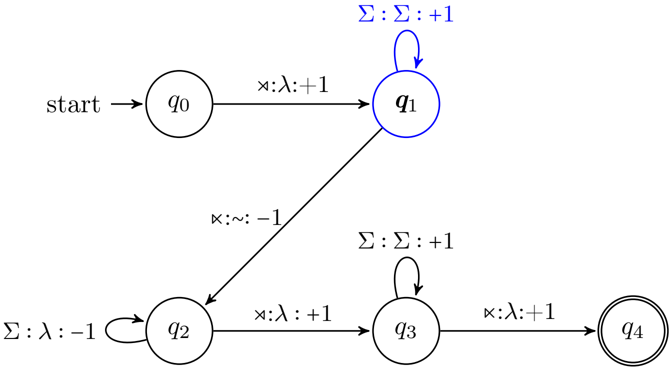
Input:	$\rtimes$	b	y	e	$\ltimes$
Output:		b			



Total reduplication with 2-way FSTs

- Indonesian example: wanita→wanita~wanita
- Working example: bye→bye~bye

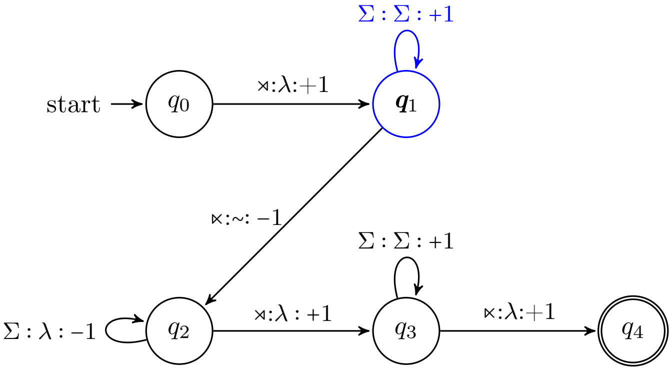
Input:	⋊	b	y	e	⋉
Output:		b	y		



Total reduplication with 2-way FSTs

- Indonesian example: wanita→wanita~wanita
- Working example: bye→bye~bye

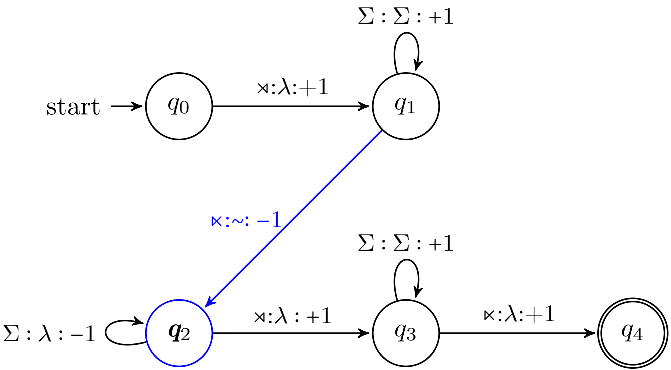
Input:	⋊	b	y	e	⋉
Output:		b	y	e	



Total reduplication with 2-way FSTs

- Indonesian example: wanita→wanita~wanita
- Working example: bye→bye~bye

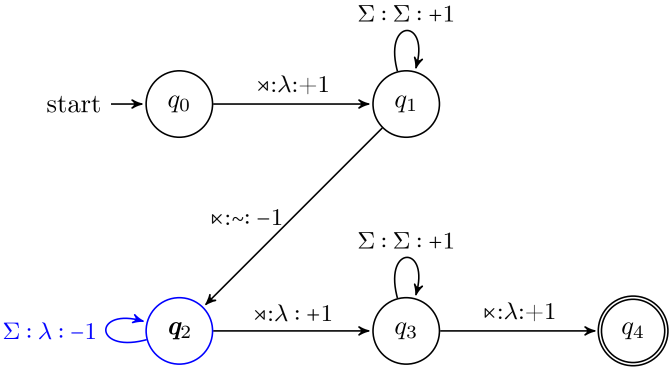
Input:	⋊	b	y	e	⋉
Output:		b	y	e	~



Total reduplication with 2-way FSTs

- Indonesian example: wanita→wanita~wanita
- Working example: bye→bye~bye

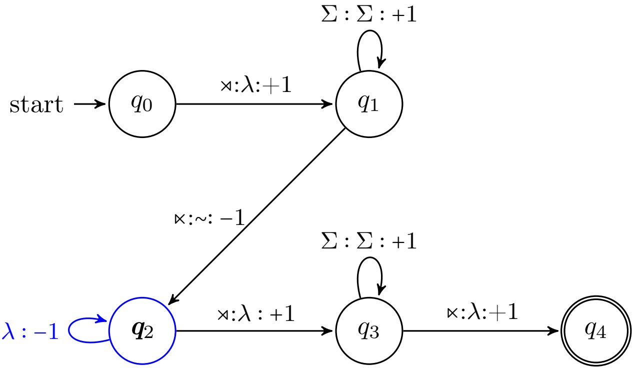
Input:	⋊	b	y	e	⋉
Output:		b	y	e	~



Total reduplication with 2-way FSTs

- Indonesian example: wanita→wanita~wanita
- Working example: bye→bye~bye

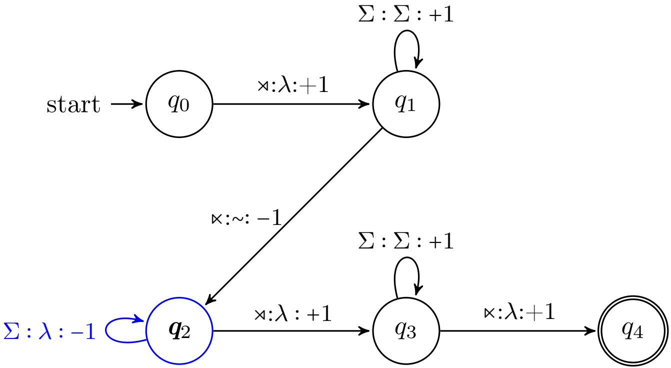
Input:	⋊	b	y	e	⋉
Output:		b	y	e	~



Total reduplication with 2-way FSTs

- Indonesian example: wanita→wanita~wanita
- Working example: bye→bye~bye

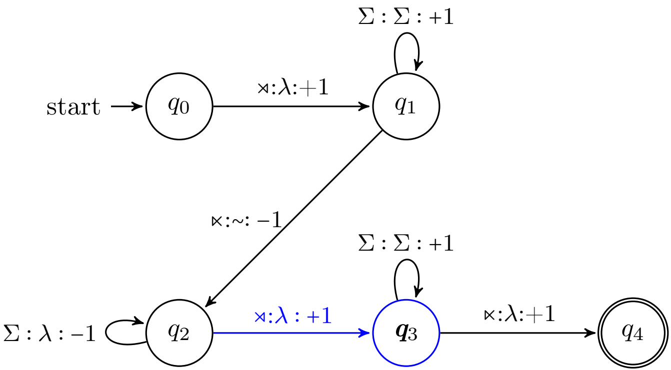
Input:	⋊	b	y	e	⋉
Output:		b	y	e	~



Total reduplication with 2-way FSTs

- Indonesian example: wanita→wanita~wanita
- Working example: bye→bye~bye

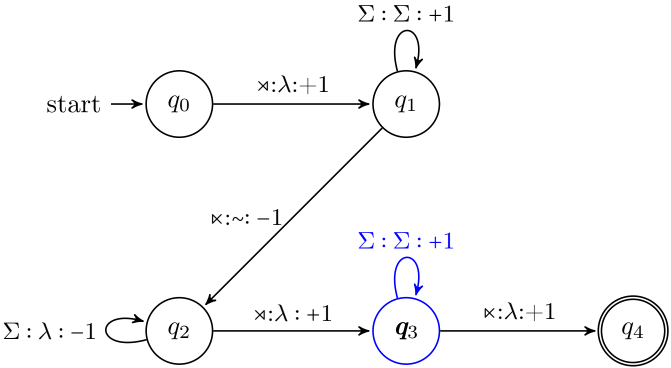
Input:	⋊	b	y	e	⋉
Output:		b	y	e	~



Total reduplication with 2-way FSTs

- Indonesian example: wanita→wanita~wanita
- Working example: bye→bye~bye

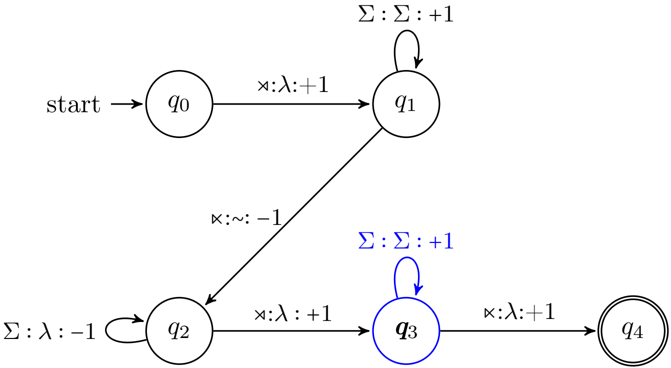
Input:	⋊	b	y	e	⋉
Output:		b	y	e	~
		b			



Total reduplication with 2-way FSTs

- Indonesian example: wanita→wanita~wanita
- Working example: bye→bye~bye

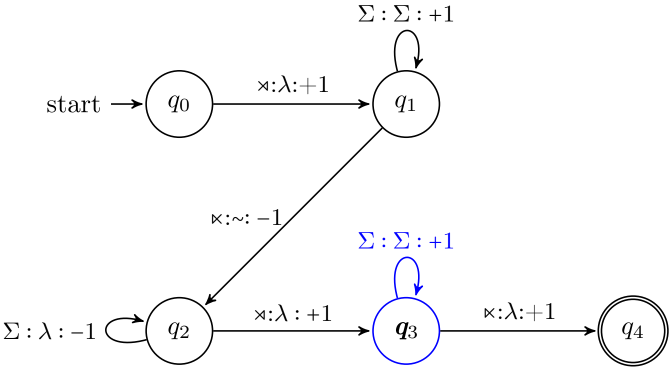
Input:	$\rtimes$	b	y	e	$\ltimes$
Output:		b	y	e	~
		b	y		



Total reduplication with 2-way FSTs

- Indonesian example: wanita→wanita~wanita
- Working example: bye→bye~bye

Input:	⋊	b	y	e	⋉
Output:		b	y	e	~
		b	y	e	



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- Indonesian example: wanita→wanita~wanita
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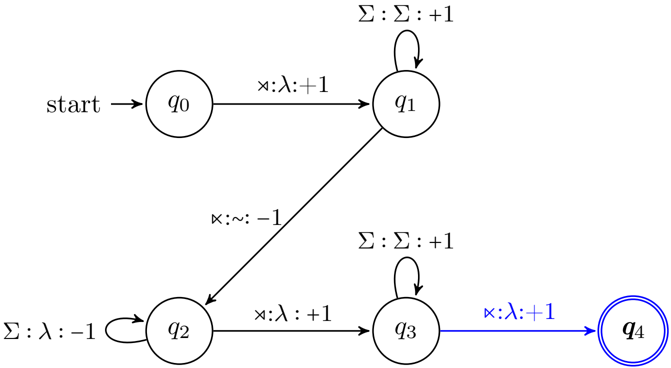
Input:	$\rtimes$	b	y	e	$\ltimes$
Output:		b	y	e	~
		b	y	e	



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- Working example: bye→bye~bye

Input:	$\rtimes$	b	y	e	$\ltimes$	
Output:		b	y	e	~	☺
		b	y	e